Journal of Physics: Conference Series

PAPER • OPEN ACCESS

Control Algorithm for Two-Tank System using Multiparametric Programming

To cite this article: A Zakaria *et al* 2023 *J. Phys.: Conf. Ser.* **2641** 012028

Control Algorithm for Two-Tank System using Multiparametric Programming

A Zakaria[1], E C Mid[1,2*], M F Mohamed[1,2], M H M Hussin[1], A S Shaari[1], E Ruslan[3], D A Hadi[3], M Masri[4]

[1]Faculty of Electrical Engineering &Technology (FKTE), Universiti Malaysia Perlis, 02600 Arau, Perlis, Malaysia
[2]Centre of Excellence for Renewable Energy (CERE), Universiti Malaysia Perlis, Kampus Pauh Putra, 02600 Arau, Perlis, Malaysia
[3]Faculty of Electrical and Electronic Engineering Technology (FTKEE), Universiti Teknikal Malaysia Melaka (UTeM), 76100 Durian Tunggal, Melaka, Malaysia
[4]Project Delivery/Project Delivery & Technology, PETRONAS, Tower 1, PETRONAS Twin Towers, Kuala Lumpur City Centre, 50088 Kuala Lumpur, Malaysia

Email: ernie@unimap.edu.my

Keywords: Two-tank System, multiparametric programming, control algorithm

Abstract. The two-tank system consists of two connected tanks, where the goal is to control the liquid levels in both tanks by manipulating the inflow rates. This study proposes a control algorithm for a two-tank system based on multiparametric programming. In this work, the flow rate is determined using multiparametric programming. The simulation studies demonstrate the proposed work's effectiveness by comparing its performance with and without noise. Results show accurate and robust regulation of liquid levels, minimizing control effort and making it suitable for systems with dynamic variations or parameter uncertainties. In conclusion, the implementation of multiparametric programming is able to estimate the value of the output for the control algorithm of the two-tank system.

1. Introduction

The two-tank system is a common control problem used in process control and optimization. It involves controlling liquid levels in two interconnected tanks [1]. The amount of fluid in the tanks must be maintained and regulated continually to achieve the intended output and meet the control objectives, such as product quality. A fundamental issue in the process industries is the regulation of liquid level in tanks and flow between tanks [2]. For instance, if liquid is not adequately regulated, it will overflow and present a safety hazard. As a result, it is necessary to control the flow between the two tanks and the tank system's liquid level.

Several papers have addressed different approaches to controlling tank systems. Barat and Mandal [3] proposed an uncertainty and disturbance estimator-based control method for liquid level control in a coupled-tank system, demonstrating better performance compared to a PID controller. In a couple tank system [4] the evaluation using PD and PID controllers are proposed. The result shows that the PID controller fulfilled all objectives related to control. But, the study did not investigate the effect of uncertainties and measurement errors on the performance of the control algorithms. In [5], the internal model controller (IMC) and optimal PID control have demonstrated efficacy in controlling the liquid level in the tanks where the settling time and overshoot of the controllers can be improved by using appropriate tuning methods. Urrea and Páez designed and compared four control strategies for an inverted conical tank system, including classical PID, gain scheduling, internal model control, and fuzzy logic [6]. The classical PID controller effectively controls liquid levels near the operation point but struggles with farther points. Modern controllers are more effective in compensating for overshoot and oscillation. This study examines contemporary plant control methods, such as control flow employing multi-parametric programming, which is anticipated to improve the system's controllability and stability.

The objective of multi-parametric programming is to attain the optimal values of decision variables that maximize a specified objective function, while satisfying constraints. These constraints can be inequalities or equalities and involve decision variables and parameters. The goal is to find the best solution that meets all the constraints while maximizing or minimizing the objective function. This approach has been applied in process system engineering [7]. It allows for considering multiple objectives and the trade-off between them, providing a holistic perspective for decision-making.

In this study, multiparametric programming is used to control flow rate. Multiparametric programming is a mathematical optimization technique that deals with problems involving multiple parameters. It allows for optimising a system over a range of parameter values rather than a single fixed value. In the context of the two-tank system, multiparametric programming can be used to find the optimal control strategy for maintaining the desired liquid levels in the tanks under various operating conditions. The multiparametric programming approach involves formulating an optimization problem considering system dynamics, constraints, and objective functions. In this case, the parameters include the operating conditions such as inflow rates, outflow rates, tank sizes, and desired liquid levels. By defining a parametric optimization problem, one can obtain a solution valid for a range of parameter values, allowing for robust control of the two-tank system.

2. Two-tank System

As shown in Figure 1, a two-tank system is proposed in this study, where the output from tank 1 is input for tank 2 [8]. The two-tank system's mathematical model is given in Equations (1) and Equation (2). For this system, height of liquid in tank 1 is given as h_1 and height of liquid in tank 2 is h_2 and flow of liquid into tank 1 is q_{in}. The parameter used in the two-tank system is given in Table 1.

$$\frac{dh_1}{dt} = -\frac{h_1}{R_1 A_1} + \frac{h_2}{R_1 A_1} + \frac{q_{in}}{A_1} \tag{1}$$

$$\frac{dh_2}{dt} = \left(\frac{1}{R_1 A_2}\right) h_1 - \frac{1}{R_2 A_2}\left(1 + \frac{R_2}{R_1}\right) h_2 \tag{2}$$

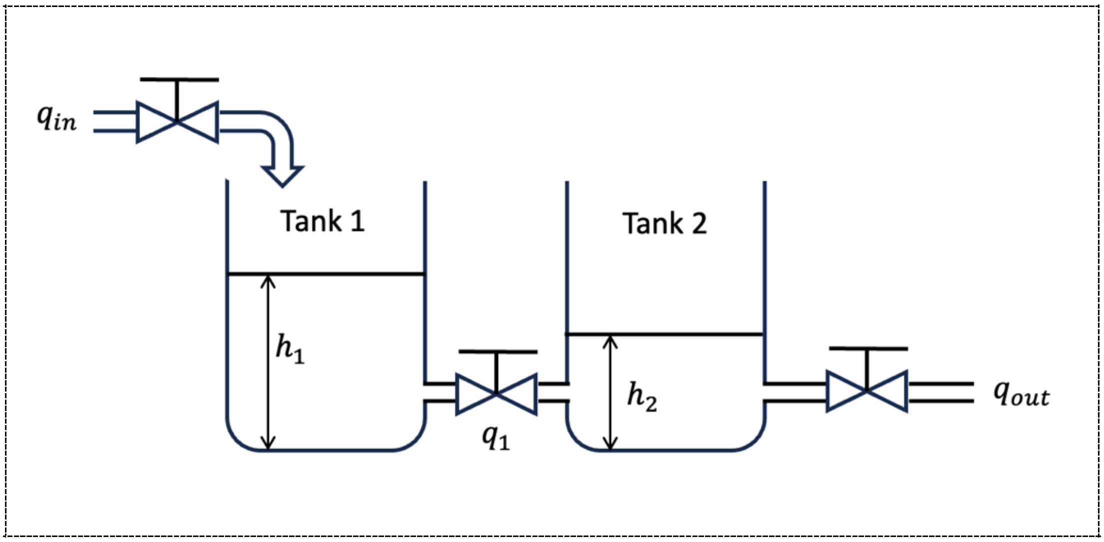

Figure 1. Two-tank system.

Table 1. Parameters of the two-tank system.

Parameter	Description	Value
A_1	Cross-sectional area of tank 1	0.01 m^2
A_2	Cross-sectional area of tank 2	0.01 m^2
R_1	Resistance parameter of valve 1 in flow line	900 s/m^2
R_2	Resistance parameter of valve 2 in flow line	360 s/m^2
q_{in}	Flow of liquid into the tank 1	5 m^3/s

3. Control algorithm for a two-tank system using multiparametric programming

This study aims to regulate the liquid levels in the second tank through multiparametric programming to manipulate the inflow and outflow valve flow rates. Hence, after the mathematical model of a two-tank system is developed, an explicit control algorithm is formulated using multiparametric programming (MPP) [9]. The objective function in this work is to minimize the error between $\boldsymbol{h_1}$ and $\boldsymbol{h_2}$ to explicitly obtain the input voltage, $\boldsymbol{q_{in}}$ as a function. The $\boldsymbol{q_{in}}$will be the control element in the two-tank system. The formulation of MPP is given below:

$$\text{Obj function } = \sum_{\theta,h_1,h_2} \{\hat{h}_1(t+1)-h_1(t+1)\}^2 + \{\hat{h}_2(t+1)-h_2(t+1)\}^2 \tag{3}$$

Subject to:

Equation (1) and Equation (2)

$$L = g + \lambda_1 x_1 + \lambda_2 x_2 \tag{4}$$

$$x_1 = h_1(t+1) = h_1(t) - \Delta k\left(\frac{h_1(t)}{R_1 A_1} + \frac{h_2(t)}{R_1 A_1} + \frac{q_{in}}{A_1}\right) = 0 \tag{5}$$

$$x_2 = h_2(t+1) = h_2(t) + \Delta k\left(\left(\frac{1}{R_1 A_2}\right)h_1(t) - \frac{1}{R_2 A_2}\left(1 + \frac{R_2}{R_1}\right)h_2(t)\right) = 0 \tag{6}$$

$$g = \left(\hat{h}_1(t+1) - h_1(t+1)\right)^2 + \left(\hat{h}_2(t+1) - h_2(t+1)\right)^2 \tag{7}$$

$$h_1(0) = 50 \tag{8}$$

$$h_2(0) = 30 \tag{9}$$

These equations are then will be solved symbolically using Mathematica software. The solution will allow us to derive $\boldsymbol{q_{in}}$, as an explicit control function and the solution is given as in Equation (10). Equation (10) is then substituted in the two-tank model, as shown in Figure 2.

$$q_{in} = \frac{\Delta k h_1(t) - \Delta k h_2(t) - A_1 h_1(t) R_1 + A_1 \hat{h}_1(t+1) R_1}{\Delta k R_1} \tag{10}$$

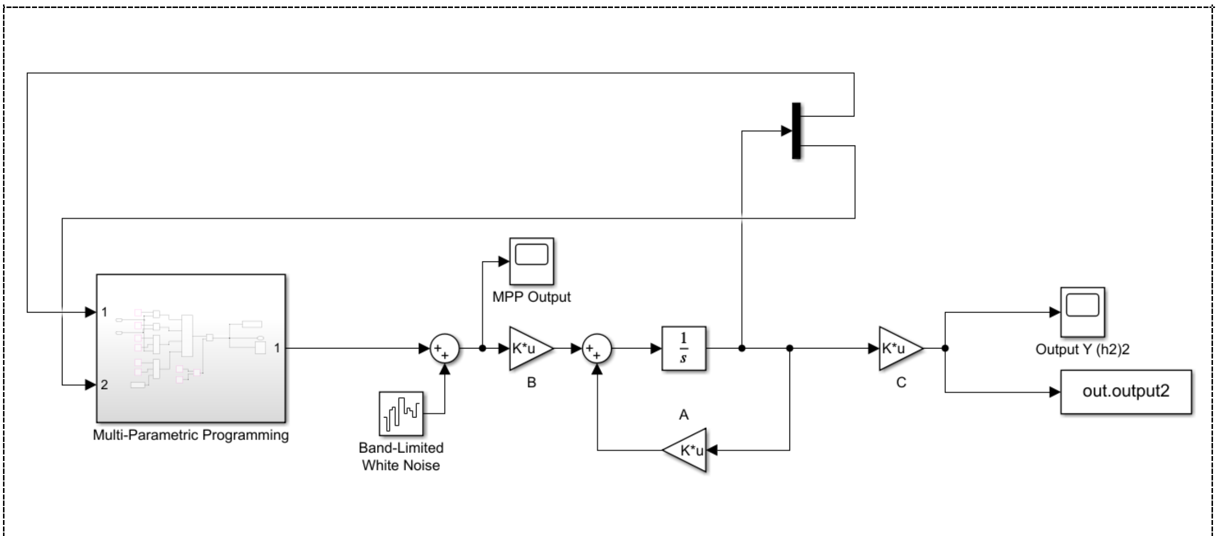

Figure 2. Simulink block of two tank system with MPP control algorithm.

4. Result and discussion

The result of simulation for q_{in} using multiparametric programming method performed without and with noise are illustrated in Figure 3 and Figure 4, correspondingly. The simulation is performed for 100 sec for the time step of 0.1. The proposed work of multiparametric programming to evaluate q_{in} using the explicit function as obtained in Equation (9) able to determine q_{in} = 5 m^3/s in without noise scenario. However, in noisy scenarios, the simulated q_{in} was influence by noise. These results show that the explicit function developed by MPP can evaluate flow of liquid into tank 1, q_{in} either noise-free or noisy scenarios.

In this work, the aim is to control the liquid levels in the tanks by manipulating the flow rates of the inlet using multiparametric programming. By implementing the explicit function as shown, the output of the two-tank system is demonstrating in Figure 5. It is observed that the output result of the two-tank system with MPP has a little decrement since the MPP system is now controlling the output. Nevertheless, the proposed work successfully controlled the two-tank system using the proposed MPP algorithm.

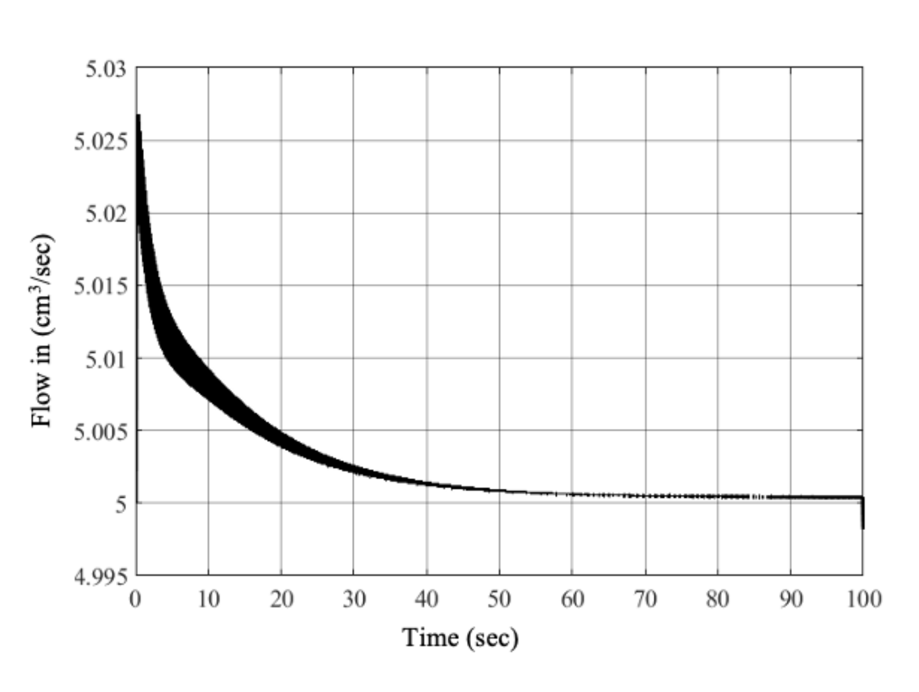

Figure 3. Response of q_{in} without noise.

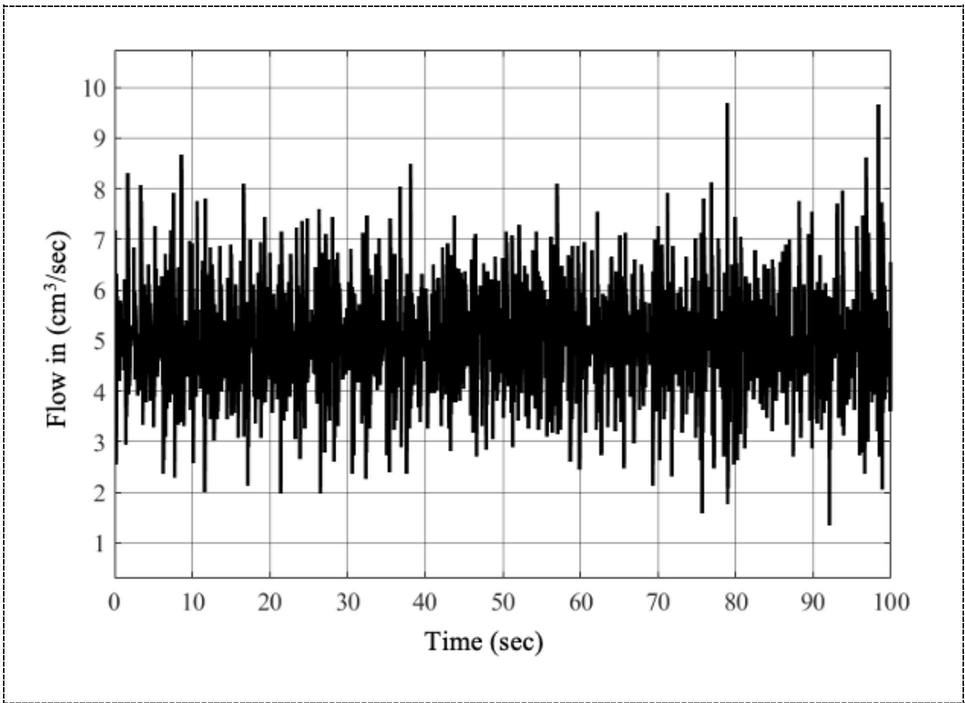

Figure 4. Response of q_{in} with noise.

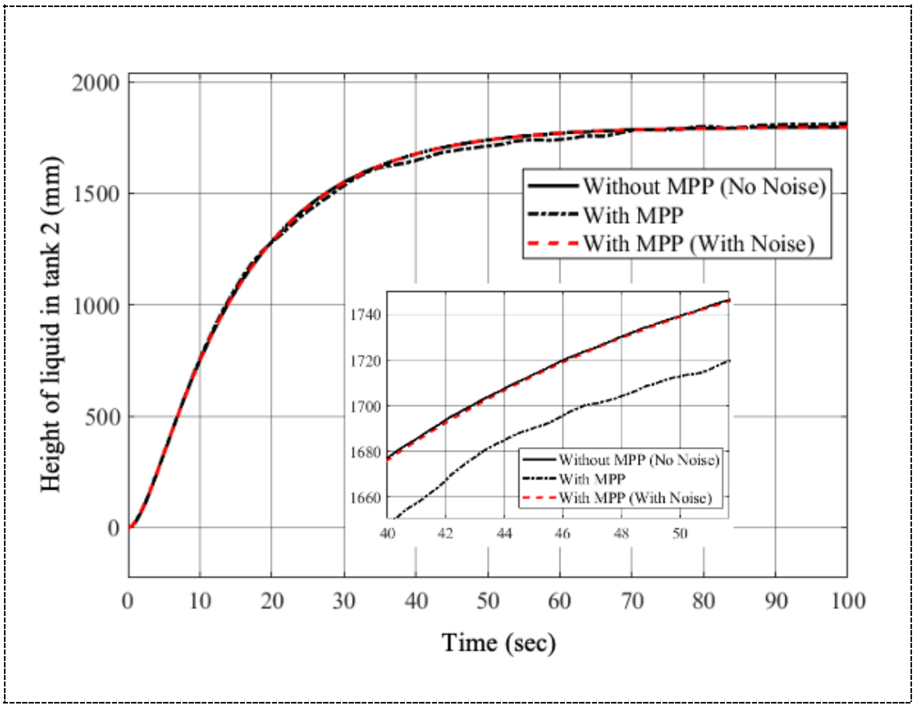

Figure 5. The output response of the two-tank system.

5. Conclusion

In this work, the two-tank system with multiparametric programming is proposed. Multiparametric programming allows simultaneously considering multiple parameters and constraints, improving performances and robustness. The proposed work successfully regulates the levels of liquid in the tanks through the utilization of multiparametric programming. The advantages of using MPP for two-tank systems include enhanced performance, better handling of constraints and improved stability. The explicit nature of the controller enables faster computation times and reduced computational complexity compared to other optimization techniques.

6. References

[1] Arce N, Piedra J, Escobar E, Lujan E, Aguilar L, Gutierrez F. 2021 Application of mathematical models to estimate the liquid level stability in a two-tank system *Proc. Int. Conf. on Computer Science Application* (Italy) Springer p 479–492

[2] Azlan Hussain M. 1999 Review of the applications of neural networks in chemical process control — simulation and online implementation *Artif. Intell. Eng.* **13** 55–68

[3] Barat P, Mandal S. 2021 Robust control of coupled-tank system using uncertainty and disturbance estimator *Proc Int. Conf. on IEEE Control, Measurement and Instrumentation* p 128–132

[4] Ayodeji O, Alexander O, Peter A, Olawale O, Frank O, Olayemi M. 2020 Model identification and level control of coupled tanks system *J. Eng. Stud. Res.* **26** 124–134

[5] Shukla S, Pati UC. 2019 Implementation of different control strategies on a quadruple tank system *Proc. Int. Conf. on Signal Processing and Integrated Networks* p. 579–83

[6] Urrea C, Páez F. 2021 Design and comparison of strategies for level control in a nonlinear tank *Process.* **9**

[7] Oberdieck R, Diangelakis NA, Nascu I, Papathanasiou MM, Sun M, Avraamidou S, 2016 On multi-parametric programming and its applications in process systems engineering. *Chem. Eng. Res. Des.* **116** 61–82
[8] Changela M, Kumar A. 2015 Designing a controller for two tank interacting system *Int. J. Sci. Res.* **4** 589–93
[9] Che Mid E, Dua V. 2017 Model-based parameter estimation for fault detection using multiparametric programming *Ind. Eng. Chem. Res*. **56** 8000–15

Acknowledgements
The authors would like to acknowledge the support from the Fundamental Research Grant Scheme (FRGS) under a grant number FRGS/1/2022/TK08/UNIMAP/02/92 from the Ministry of Higher Education (MOHE) Malaysia and University Malaysia Perlis.